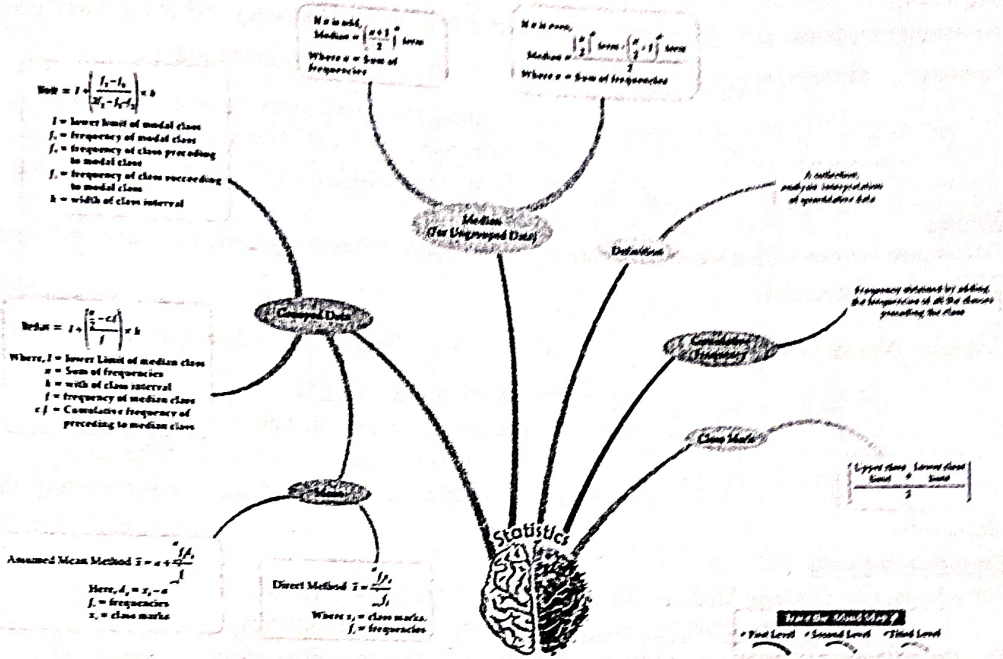
CHAPTER 13
STATISTICS
MIND MAP



GIST OF THE CHAPTER

* In the chapter of statistics, we will study about the techniques for finding the mean, median and mode of grouped data.

* There are three measures of central tendency which are i) mean ii) mode iii) median

Mean (the average of given set of data)

To calculate the mean of grouped data, there are three methods

(1) Direct method (2)Assumed mean method (shortcut method) (3) Step deviation method

i) **Direct method**

a. Make a frequency table in which the first column consists of the class intervals, the second consists of the corresponding frequencies (fi), the third consists of the class marks (mid point of the class intervals) denoted by xi and the fourth column is $fixi$

1. Formula for mean in direct method $=\underline{x}=\frac{\Sigma fixi}{\Sigma fi}$

ii) **Assumed mean method (short-cut method)**

1. Formula Mean $(\underline{x})=a+\frac{\Sigma fidi}{\Sigma fi}$
2. $di = xi - a$ where 'a' is assumed mean (one of the mid values)
3. di maybe -ve (or) +ve (or) zero

iii) **Step deviation method**

1. Formula Mean $(\underline{x})= a + \left(\frac{\Sigma fiui}{\Sigma fi}\right) Xh$

Here $ui = \frac{xi-a}{h}$, where 'a' is assumed mean,

h is the class size (upper limit-lower limit)

Mode

In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies.

To compute the mode, at first, we locate a class with the maximum frequency, called the modal class

Formula: $\text{Mode} = l + \left(\frac{f1-f0}{2f1-f0-f2}\right) X\ h$ Where l= lower limit of the modal class

$f1$= frequency of modal class(the highest frequency)

$f0$= frequency of the class preceding the modal class

$f2$= frequency of the class succeeding the modal class

Median

To compute median, at first we locate a class whose cumulative frequency is greater than or nearest to $\frac{n}{2}$, called the median class.

Formula: $\text{Median}=l + \left(\frac{\frac{n}{2}-cf}{f}\right) X\ h$ Where l= lower limit of the modal class

n= sum of frequency $=\sum fl$

f= frequency of the median class

h= size of the median class (class size to be equal)

cf=cumulative frequency of the class preceding the median class

Empirical formula

The relation among Mean, Mode and Median: Mode = 3 Median – 2 Mean

MULTIPLE CHOICE QUESTIONS (1 MARK QUESTIONS)

1) If the difference of the mode and median of a data is 24, then the difference of its median and mean is

(a)8 (b) 12 (c) 24 (d) 36

Ans: (b)

Solution: Mode-median=24(given)

Mode =3 median-2 mean

Mode – median =2(median-mean)

$\Rightarrow$ Median-mean $= \frac{24}{2} = 12$

2) If the value of each observation in the data is increased by 2, then median of the new data is

(a)Increased by 2 (b) Increased by 2n (c) Remains same (d) Decreases by 2

Ans: (a)

Solution: As we know, if the value of each observation in the data is increased, then the median will also increase. Here, each observation in the data is increased by 2. Hence the new median will also increase by 2

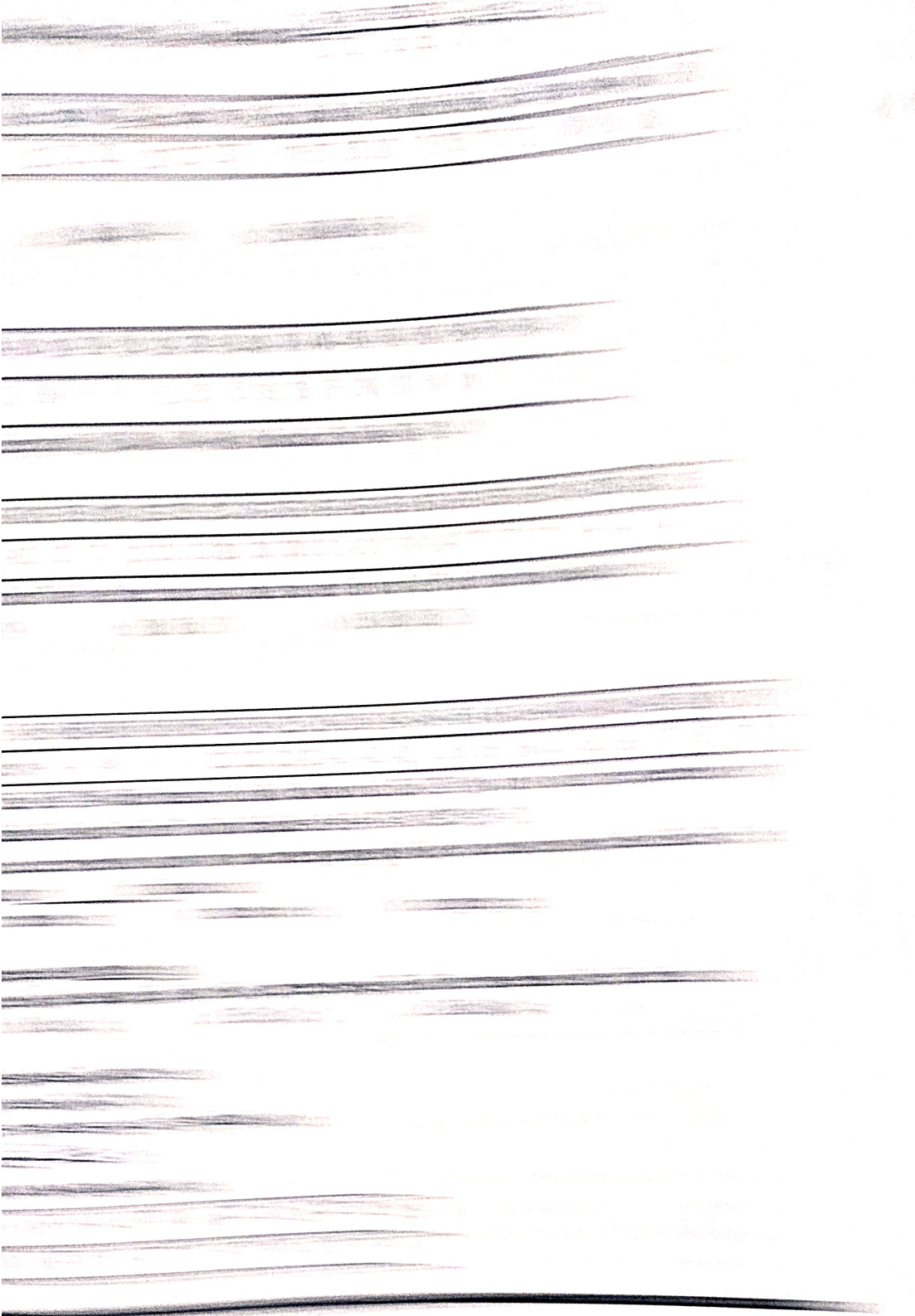
3) The following distribution gives the daily income of 50 workers of the factory. The lower limit of the modal class is

Income (in ₹)	400-424	425-449	450-474	475-499	500-524
Number of workers	12	14	8	6	10

(a)25 (b) 449 (c) 424.5 (d) 425.5

Ans: (c)

Solution: C.I against highest frequency (14) is 425-449 (Modal class), but the class is not continuous. ∴ 424.5-449.5. The lower limit is 424.5



4)The class marks of the median class of the following data is

Class intervals	10-25	25-40	40-55	55-70	70-85	85-100
frequency	2	3	7	6	6	6

(a) 40 (b) 55 (c) 47.5 (d) 62.5

Ans: (d)

Solution: n=30 $\Rightarrow \frac{n}{2}$ =15 . Cumulative frequencies are 2,5,12,18,24,30

$\frac{n}{2}$ lies within 18 → against to 55-70 ;(Median class). So, Class marks is $\frac{51+70}{2} = \frac{125}{2} = 62.5$

5)For the following distribution:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
Number of students	3	12	27	57	75	80

The modal class is:

(a) 10-20 (b) 20-30 (c) 30-40 (d) 50-60

Ans: (c)

Solution:

Class intervals (C.I)	0-10	10- 20	20- 30	30-40	40- 50	50-60
frequency	3	9	15	30	18	5

Now C.I against highest frequency (30) is 30-40 ∴ Modal class is 30-40

6)Consider the data:

Class intervals	65-85	85-105	105-125	125-145	145-165	165-185	185-205
frequency	4	5	13	20	14	7	4

The difference of the upper limit of the median class and the lower limit of the modal class:

(a) 0 (b) 19 (c) 38 (d) 20

Ans: (d)

Solution:

Class intervals	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Cumulative frequency	4	9	22	42	56	63	67

n/2 =67/2 ⇒ 33.5 lies in 125-145 (Median class), Modal class is 125-145 (as of its high frequency

20) Upper limit of Median class-Lower limit of modal class = 145- 125 =20

7)After an examination, a teacher wants to know the average marks of the students in the class. She requires the calculation _____of marks.

(a)Median (b) mean (c) mode (d) range

Ans: (b)

Solution: Average related to mean.

8)The mode and mean of the data are $15x$ and $18x$, respectively. Then the median of the data is

(a) x (b) $11x$ (c) $17x$ (d) $34x$

Ans: (c)

Solution: mode=3median-2 mean

$15x$=3 median-2 ($18x$)

$15x+36x$=3 median ⇒ $51x$=3 median

Median= $\frac{51x}{3} = 17x$

9) The mode of the following data is

xi	10	14	18	21	25
fi	10	15	7	9	9

(a)16 (b) 10 (c) 12 (d)14

Ans: (d)

Solution: The mode is the most frequent observation. Here, the mode is 14 with a frequency of 15

10)The mean of the following distribution is

x_i	12	14	18	20
fi	3	5	8	7

(a)19.5 (b)18 (c)16.95 (d)15.24

Ans: (c)

Solution: $\sum fixi$=3x12+5x14+8x18+7x20 =36+70+144+140=390

Mean= $\frac{\sum fixi}{\sum fi} = \frac{390}{23} = 16.95$

ASSERTION AND REASONING QUESTIONS

DIRECTION: In the following questions, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

(a) Both assertion **(A)** and reason **(R)** are true and reason **(R)** is the correct explanation of assertion **(A)**

(b) Both assertion **(A)** and reason **(R)** are true and reason **(R)** is not the correct explanation of assertion **(A)**

(c) Assertion **(A)** is true but reason **(R)** is false.

(d) Assertion **(A)** is false but reason **(R)** is true.

1. **Assertion (A):** The arithmetic mean of the following given frequency distribution table is 13.81.

Marks	2.5 – 5.5	5.5 – 8.5	8.5 – 11.5	11.5 – 14.5	14.5 – 17.5	17.5 – 20.5
No. of Students	7	10	15	20	25	30

Reason (R): Mean = $\sum fx / \sum f$

Ans: (a)

2. **Assertion (A):** If the value of mode and mean is 60 and 66 respectively, then the value of median is 64.

Reason (R): Median = (mode + 2 mean)/2

Ans: (c)

$Median = \frac{1}{3}(mode + 2mean) = \frac{1}{3}[60 + 2(66)] = \frac{1}{3} \times 192 = 64$

3. **Assertion(A):** If the value of mode and median is 50.5 and 45.5 respectively, then the value of 2 mean is 86.

Reason (R): Median = (Mode + 2 Mean)

Ans.(c)

Mode=3Median – 2Mean=>(50.5) = 3 (45.5) – 2 Mean => 2Mean = 136.5 –50.5 = 86

4. **Assertion (A):** Consider the following frequency distribution:

Class Interval	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency	5	9	12	6	8

The modal class is 10 – 15.

Reason (R): The class having maximum frequency is called the modal class.

Ans. (d)

The maximum frequency is 12, which lies in the interval 20 – 25. So, the modal class is 20 – 25.

5. **Assertion (A):** For a moderately asymmetric distribution, Mode- Median = 2(Median-Mean)

Reason (R): For a symmetric distribution Mean = Median = Mode

Ans(b)

6. **Assertion (A):** The algebraic sum of the deviations of a frequency distribution from its mean is 0
Reason (R): Mode of a frequency distribution cannot be determined graphically.
Ans(c)

7. **Assertion (A):** The mean of 1, 4, 7, 10 .....301 is 151.
Reason (R): The mean of series a, a+d, a+2d.....a+2nd, is a+nd
Ans(b)

8. **Assertion (A) :** The mode of the following distribution is 52.

Class interval	0-20	20-40	40-60	60-80
Frequency	4	3	2	2

Reason (R) : The value of the observation which occurs most often is the mode.

Ans:(d)

9. **Assertion (A):** If for a data mean: median = 9:8 then Median : Mode = 4:3
Reason (R): Emperical formula mode = 3 median – 2 Mean
Ans(a)

10. **Assertion (A):** The arithmetic mean of 1002, 1004, 1006, 1008, 1003 and 1007 is 1005
Reason (R): arithmetic mean= $\frac{\text{Sum of all observations}}{\text{Total no. of observations}}$
Ans:(a)

VERY SHORT ANSWER TYPE QUESTIONS(2 MARKS QUESTIONS)

1. The mean and mode of a frequency distribution are 28 and 16 respectively. Find the median.
Ans:We know that, Mode = 3 Median – 2 Mean
$\Rightarrow$ 3 Median = Mode + 2 Mean $\Rightarrow$ 3 Median = 16 + 2 × 28 $\Rightarrow$ Median = 72/3 = 24

2. The runs scored by a batsman in 35 different matches are given below:

Runs Scored	0-15	15-30	30-45	45-60	60-75	75-90
Frequency	5	7	4	8	8	3

Find the lower limit of the median class .
Ans:

Runs Scored	Frequency	*cf*
0-15	5	5
15-30	7	12
30-45	4	16
45-60	8	24
60-75	8	32
75-90	3	35

Here, n = 35 $\Rightarrow$ n/2 = 17.5 & Median class is 45 – 60. Hence, lower limit is 45.

3.For the following distribution: Find the modal class.

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of Students	3	12	27	57	75	80

Ans:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of Students	3	9	15	30	18	5

Modal class is 30 – 40

4. For the following distribution, find the upper limit of the median class

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

Ans: Here, $n = 57$ So, $\frac{n}{2} = 28.5$

Class	Frequency	cf
$-0.5 - 5.5$	13	13
$5.5 - 11.5$	10	23
$11.5 - 17.5$	15	38
$17.5 - 23.5$	8	46
$23.5 - 29.5$	11	57

The cumulative frequency, just greater than 28.5, is 38 which belongs to class $11.5 - 17.5$. So, the median class is $11.5 - 17.5$. Its upper limit is17.5

5. If the mean of the following distribution is 2.6, then y = ?

Variable (x)	1	2	3	4	5
Frequency	4	5	y	1	2

Ans:

Variable (x)	Frequency (f)	fx
1	4	4
2	5	10
3	y	3y
4	1	4
5	2	10
Total	$y + 12$	$3y + 28$

Here, $\sum f = y + 12$ and $\sum fx = 3y + 28$

$$Mean, \bar{x} = \frac{\sum fx}{\sum f} \Rightarrow 2.6 = \frac{3y+28}{y+12} \Rightarrow 3y+28 = 2.6y+31.2$$

$$\Rightarrow 0.4y = 3.2 \Rightarrow y = 8$$

6.In a continuous frequency distribution with usual notations, if $l = 32.5, f_1 = 15, f_0 = 12, f_2 = 8$ and $h = 8$, then find the mode.

Ans.
$$Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$\Rightarrow Mode = 32.5 + \frac{15-12}{30-12-8} \times 8 = 32.5 + \frac{3}{10} \times 8 = 32.5 + 2.5 = 34.9$$

7.Consider the following frequency distribution of the heights (in cm) of 60 students of a class:

Class	150 – 155	155 – 160	160 – 165	165 – 170	170 – 175	175 – 180
Frequency	15	13	10	8	9	5

Find the upper limit of the median class.

Ans.

Class	Frequency	*cf*
150 – 155	15	15
155 – 160	13	28
160 – 165	10	38
165 – 170	8	46
170 – 175	9	55
175 – 180	5	60

Here, $n = 60 \Rightarrow \frac{n}{2} = 30$, Median class is 160 – 165, Hence, upper limit is 165.

8. If the value of each observation of a statistical data is increased by 3, then the what happens to mean of the data?

Ans. Mean increases by 3

If each value of observation is increased by 3, then mean is also increased by 3.

9.Consider the following frequency distribution of the heights (in cm) of 60 students of a class:

Class	150 – 155	155 – 160	160 – 165	165 – 170	170 – 175	175 – 180
Frequency	16	12	9	7	10	6

Find the sum of the lower limit of the modal class and the upper limit of the median class.

Ans:

Class	Frequency	*cf*
150 – 155	16	16
155 – 160	12	28
160 – 165	9	37
165 – 170	7	44
170 – 175	10	54
175 – 180	6	60

The class having the maximum frequency is the modal class.

So, the modal class is 150 – 155 and its lower limit is 150.

Also, $n = 60 \Rightarrow n/2 = 30$, Median class is 160 – 165 whose upper limit is 165

$\therefore$ Required sum = (150 + 165) = 315

10.If the difference of Mode and Median of a data is 24, then find the difference of median and mean.

Ans: mode – median = 24 (given)

$\therefore$ mode = 24 + median

Since, mode = 3 median – 2 mean [By empirical relation]

$\therefore$ 24 + median = 3 median – 2 mean

$\Rightarrow$ 2 median – 2 mean = 24

$\Rightarrow$ median – mean = 12

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

1.If mode of the following frequency distribution is 55 then find the value of x.

Class	0 – 15	15 – 30	30 – 45	45 – 60	60 – 75	75 – 90
Frequency	10	7	*x*	15	10	12

Ans: Since the mode is 55 which belongs to 45 – 60, therefore modal class is 45 – 60

Here, $l = 45, f_0 = x, f_1 = 15, f_2 = 10, h = 15$

$$Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \Rightarrow 55 = 45 + \frac{15 - x}{30 - x - 10} \times 15$$

$$\Rightarrow 10 = \frac{15 - x}{20 - x} \times 15 \Rightarrow 2 = \frac{15 - x}{20 - x} \times 3 \Rightarrow 40 - 2x = 45 - 3x$$

$$\Rightarrow 30 - 2x = 45 - 40 \Rightarrow x = 5$$

2. The mode of a grouped frequency distribution is 75 and the modal class is 65-80. The frequency of the class preceding the modal class is 6 and the frequency of the class succeeding the modal class is 8. Find the frequency of the modal class.

Ans. Here, $l = 65, f_0 = 6, f_1 = x, f_2 = 8, h = 15$

$$Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \Rightarrow 75 = 65 + \frac{x - 6}{2x - 6 - 8} \times 15$$

$$\Rightarrow 10 = \frac{x - 6}{2x - 14} \times 15 \Rightarrow 2 = \frac{x - 6}{2x - 14} \times 3 \Rightarrow 4x - 28 = 3x - 18$$

$$\Rightarrow 4x - 3x = 28 - 18 \Rightarrow x = 10$$

3. If the mean of the following frequency distribution is 62.8, then find the missing frequency x :

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	5	8	x	12	7	8

Ans.

Class	Frequency	x	fx
0 – 20	5	10	50
20 – 40	8	30	240
40 – 60	x	50	$50x$
60 – 80	12	70	840
80 – 100	7	90	630
100 – 120	8	110	880
	$x + 40$		$50x + 2640$

Here, $\sum f = x + 40$ and $\sum fx = 50x + 2640$

$$Mean, \bar{x} = \frac{\sum fx}{\sum f} \Rightarrow 62.8 = \frac{50x + 2640}{x + 40} \Rightarrow 2512 + 62.8x = 50x + 2640$$

$\Rightarrow 62.8x - 50x = 2640 - 2512 \Rightarrow 12.8x = 128 \Rightarrow x = 10$ $\therefore$ Missing frequency, $x = 10$

4. Calculate median marks of the following data:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	Total
No. of Students	8	16	36	34	6	100

Ans:

Marks	No. of Students	cf
0 – 10	8	8
10 – 20	16	24
20 – 30	36	60
30 – 40	34	94
40 – 50	6	100
Total	100	

Here, n = 100 ⇒ n/2 = 50

⇒ Median class is 20 – 30

$l = 20, cf = 24, f = 36, h = 10$

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\Rightarrow Median = 20 + \left(\frac{50-24}{36}\right) \times 10 = 20 + \frac{26 \times 10}{36} = 20 + \frac{65}{9} = 20 + 7.22 = 27.22$$

5. The arithmetic mean of the following frequency distribution is 53. Find the value of k.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	12	15	32	k	13

Ans:

Class	Frequency	x	u	fu
0 – 20	12	10	–3	–36
20 – 40	15	30	–2	–30
40 – 60	32	50	–1	–32
60 – 80	k	70	0	0
80 – 100	13	90	1	13
Total	$k + 72$			–85

Here, $\sum f = k + 72$ and $\sum fu = -85$, $h = 20$, $a = 70$

$$Mean, \bar{x} = a + \left(\frac{\sum fu}{\sum f} \times h\right) \Rightarrow 53 = 70 + \left(\frac{-85}{k+72} \times 20\right) \Rightarrow -17 = \frac{-85 \times 20}{k+72} \Rightarrow 1 = \frac{100}{k+72}$$

$$\Rightarrow k + 72 = 100 \Rightarrow k = 100 - 72 = 28$$

6. The below table shows the ages of persons who visited a museum on a certain day. Find the median age of the person visiting the museum.

Age (inyears)	Less than10	Less than20	Less than30	Less than40	Less than50	Less than60
No. of persons	3	10	22	40	54	71

Ans:

Age (in years)	No. of persons	cf
0 – 10	3	3
10 – 20	7	10
20 – 30	12	22
30 – 40	18	40
40 – 50	14	54
50 – 60	17	71

Here, $n = 71 \Rightarrow n/2 = 35.5$

$\Rightarrow$ Median class is 30 – 40

$l = 30,\ cf = 22,\ f = 18,\ h = 10$

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\Rightarrow Median = 30 + \left(\frac{35.5 - 22}{18}\right) \times 10 = 30 + \frac{13.5 \times 10}{18} = 30 + \frac{135}{18} = 30 + 7.5 = 37.5$$

The median age of the person visiting the museum is 37.5 years.

7.Heights of 50 students in class X of a school are recorded and following data is obtained:

Height (in cm)	130 – 135	135 – 140	140 – 145	145 – 150	150 – 155	155 – 160
No. of students	4	11	12	7	10	6

Find the median height of the students.

Ans: 144.17

8.Calculate mode of the following data:

Marks	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
No. of Students	5	10	12	6	3

Ans: Since the maximum frequency is 12 which belongs to 40 – 60, therefore modal class is 40 – 60

Here, $l = 40,\ f_0 = 10,\ f_1 = 12,\ f_2 = 6,\ h = 20$

$$Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \Rightarrow Mode = 40 + \frac{12 - 10}{24 - 10 - 6} \times 20 = 40 + \frac{2}{8} \times 20 = 40 + 5 = 45$$

9.Calculate median marks of the following data:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of Students	2	12	22	8	6

Ans: 25

10. Calculate mode of the following data:

Marks	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
No. of Students	7	5	10	12	6

Ans: 19.5

LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1. The median of the following data is 868. Find the values of x and y, if the total frequency is 100

Class	800 – 820	820 – 840	840 – 860	860 – 880	880 – 900	900 – 920	920 – 940
Frequency	7	14	x	25	y	10	5

Ans:

Class	Frequency	C.f
800 – 820	7	7
820 – 840	14	21
840 – 860	x	x + 21
860 – 880	25	x + 46
880 – 900	y	x + y + 46
900 – 920	10	x + y + 56
920 – 940	5	x + y + 61

From table, we have $x + y + 61 = 100 \Rightarrow x + y = 100 - 61 \Rightarrow x + y = 39$

Here, median = 868, therefore median class is 860 – 880

So, $l = 860$, cf = x + 21, f = 25, h = 20, n/2 = 50

Now, $Median = l + \left(\frac{\frac{n}{2} - cf}{f} \times h\right) \Rightarrow 868 = 860 + \left(\frac{50 - (x+21)}{25} \times 20\right)$

$\Rightarrow 868 - 860 = \left(\frac{50 - x - 21)}{5} \times 4\right) \Rightarrow 8 = \frac{29 - x}{5} \times 4$

$\Rightarrow 40 = (29 - x)4 \Rightarrow 29 - x = 10 \Rightarrow x = 29 - 10 = 19$

$\Rightarrow y = 39 - 19 = 20$

2. The distribution below gives the makes of 100 students of a class, if the median makes are 24, find the frequencies f_1 and f_2

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
No. of students	4	6	10	f_1	25	f_2	18	5

Ans:

Class	Frequency	cf
0 – 5	4	4
5 – 10	6	10
10 – 15	10	20
15 – 20	f_1	$20 + f_1$
20 – 25	25	$45 + f_1$
25 – 30	f_2	$45 + f_1 + f_2$
30 – 35	18	$63 + f_1 + f_2$
35 – 40	5	$68 + f_1 + f_2$

Now, Median = 24 (Given) , So, median class = 20 – 25

$l = 20$, $h = 5$, $n/2 = 50$, $cf = 20 + f_1$, $f = 25$, We know, Median $= l + \left(\frac{\frac{n}{2} - cf}{f} \times h\right)$

$\Rightarrow 24 = 20 + \frac{50 - (20 + f_1)}{25} \times 5 \Rightarrow 4 = \frac{30 - f_1}{5} \Rightarrow 30 - f_1 = 20 \Rightarrow f_1 = 10$,

Also, sum of frequencies $= 100$

$\Rightarrow 68 + f_1 + f_2 = 100 \Rightarrow f_1 + f_2 = 32 \Rightarrow 10 + f_2 = 32 \Rightarrow f_2 = 22$

$\therefore f_1 = 10$, $f_2 = 22$.

3. The distribution below gives the marks of 40 students of a class, if the median marks are 32.5, find the frequencies f_1 and f_2

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
No. of students	f_1	5	9	12	f_2	3	2	40

Ans.

Marks	No. of students	cf
0-10	f_1	f_1
10-20	5	$5 + f_1$
20-30	9	$14 + f_1$
30-40	12	$26 + f_1$
40-50	f_2	$26 + f_1 + f_2$
50-60	3	$29 + f_1 + f_2$
60-70	2	$31 + f_1 + f_2$
Total	40	

Here, $n = 40 \Rightarrow 31 + f_1 + f_2 = 40 \Rightarrow f_1 + f_2 = 9$ ... (i)

Given, median = 32.5, which lies in the class interval 30-40. So, median class is 30-40.

$\therefore l = 30$, $h = 10$, $f = 12$, $n = 40$ and c.f. of preceding class, $cf = f_1 + 14$

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\Rightarrow 32.5 = 30 + \left(\frac{20 - f_1 - 14}{12}\right) \times 10 \Rightarrow 2.5 = \left(\frac{6 - f_1}{12}\right) \times 10 \Rightarrow 30 = (6 - f_1) \times 10 \Rightarrow 3 = 6 - f_1 \Rightarrow f_1 = 3$$

$$\Rightarrow f_2 = 9 - 3 = 6$$

4.The mean of the following data is 42. Find the missing frequencies x and y if the sum of frequencies is 100.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	10	x	13	y	10	14	9

Ans.

Class	Frequency	x	u	fu
0-10	7	5	-2	-14
10-20	10	15	-1	-10
20-30	x	25	0	0
30-40	13	35	1	13
40-50	y	45	2	2y
50-60	10	55	3	30
60-70	14	65	4	56
70-80	9	75	5	45
Total	100			2y + 120

Here, $\sum f = 100 = x + y + 63 \Rightarrow x + y = 37$

and $\sum fu = 2y + 120$, $h = 10$, $a = 25$

$$Mean, \bar{x} = a + \left(\frac{\sum fu}{\sum f} \times h\right) \Rightarrow 42 = 25 + \left(\frac{2y+120}{100} \times 10\right) \Rightarrow 17 = \frac{2y+120}{10} \Rightarrow 170 = 2y + 120$$

$$\Rightarrow 2y = 170 - 120 = 50 \Rightarrow y = 25 \Rightarrow x = 37 - 25 = 12$$

5.The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure.

Expenditure	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Number of families	24	40	33	28	30	22	16	7

Ans:

Expenditure	Number of families
1000-1500	24
1500-2000	40
2000-2500	33
2500-3000	28
3000-3500	30
3500-4000	22
4000-4500	16
4500-5000	7

Modal class = 1500-2000, l = 1500, Frequencies: $f_1 = 40$, $f_0 = 24$, $f_2 = 33$ and $h = 500$

Mode formula:

Mode = $l + [(f_1 - f_0)/(2f_1 - f_0 - f_2)] \times h$

Substitute the values in the formula, we get

Mode = $1500 + [(40 - 24)/(80 - 24 - 33)] \times 500$, Mode = $1500 + [(16 \times 500)/23]$

Mode = $1500 + (8000/23) = 1500 + 347.83$ = Rs.1847.83

Class Interval	fi	xi	di=xi-a	ui=di/h	fiui
1000-1500	24	1250	-1500	-3	-72
1500-2000	40	1750	-1000	-2	-80
2000-2500	33	2250	-500	-1	-33
2500-3000	28	2750=a	0	0	0
3000-3500	30	3250	500	1	30
3500-4000	22	3750	1000	2	44
4000-4500	16	4250	1500	3	48
4500-5000	7	4750	2000	4	28
Total	$\sum fi=200$				$\sum fiui=-35$

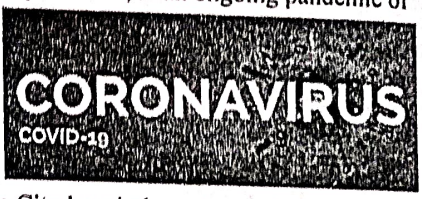
Mean = $\bar{x} = a + (\sum fiui / \sum fi) \times h$

Substitute the values in the given formula

$= 2750 + (-35/200) \times 500 \Rightarrow 2750 - 87.50 \Rightarrow 2662.50$

CASE BASED QUESTIONS (4 MARKS QUESTIONS)

1)The COVID-19 pandemic, also known as the coronavirus pandemic, is an ongoing pandemic of coronavirus disease 2019 (COVID-19) caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2). It was first identified in December 2019 in Wuhan, China. During survey, the ages of 80 patients infected by COVID and admitted in the one of the City hospital were recorded and the collected data is represented in the less than cumulative frequency distribution table.



Age(in year)	Below 15	Below 25	Below 35	Below 45	Below 55	Below 65
No. of patients	6	17	38	61	75	80

Based on the above information, answer the following questions.

(i) Find the modal class interval.

(ii) Find the median class interval

(iii)(a) Find the modal age of the patients admitted in the hospital.

(OR)

(iii)(b) Find the median age of the patients admitted in the hospital.

Ans:

2.Overweight and obesity may increase the risk of many health problems, including diabetes, heart disease, and certain cancers. The basic reason behind is the laziness, eating more junk foods and less physical exercise. The school management give instruction to the school to collect the weight data of each student. During medical check of 35 students from Class X- A, there weight was recorded as follows:

(i) Find the median class of the given data.

(ii) Find the modal class of the given data.

(iii)(a) Calculate the median weight of the given data.

(OR)

(iii)(b) Find the mean of the given data.

Weight (in kg)	Less than 38	Less than 40	Less than 42	Less than 44	Less than 46	Less than 48	Less than 50	Less than 52
No. of Students	0	3	5	9	14	28	32	35

Ans:

3.India meteorological department observe seasonal and annual rainfall every year in different subdivisions of our country. It helps them to compare and analyze the results. The table given below shows sub-division wise seasonal (monsoon) rainfall (mm) in 2018

Rainfall (in mm)	200 – 400	400 – 600	600 – 800	800 -1000	1000-1200	1200-1400	1400-1600	1600-1800
Number of Sub-divisions	2	4	7	4	2	3	1	1

Based on the above information, answer the following questions.

(i) Write the modal class.

(ii)(a) Find the median of the given data.

(OR)

(ii)(b) Find the mean rainfall in this season.

(iii)If sub-division having at least 1000 mm rainfall during monsoon season, is considered good rainfall sub-division, then how many sub-divisions had good rainfall?

Ans:

HIGHER ORDER THINKING SKILLS

1.The average weight of A, B, C is 45 kg. If the average weight of A and B be 40 kg. and that of B and C be 43 kg, find the weight of B.

Ans: Weight of B = (A + B)'s weight + (B + C)'s weight – (A + B + C)'s weight

= 2(40 kg)+ 2(43 kg) –3(45 kg) = 80 + 86 – 135 = 31 kg

2.The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Ans: x = 9 and y = 15

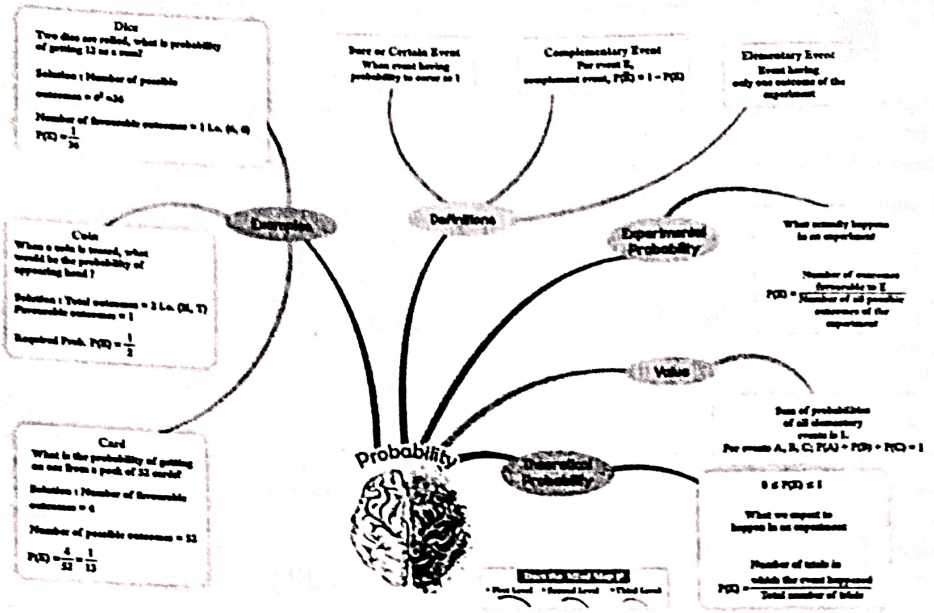
Class interval	0 -100	100 -200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Frequency	2	5	x	12	17	20	y	9	7	4

3.A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are only given to persons having age 18 years onwards but less than 60 years.

Age (in years)	Number of policyholders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Ans:35.76 Years

CHAPTER 14
PROBABILITY
MIND MAP



Gist of the lesson:The word 'Probability' is commonly used in our day-to-day conversation and we generally use this word even without going into details of its actual meaning. In general, people have a rough idea about its meaning.Some of the statements like:

- Probably it may rain today
- He/she may possibly join politics
- She is probably right

Probability numerically measures the degree of certainty of the occurrence of events.

Experiments and its outcomes

An operation which can produce some well-defined outcomes is called an experiment and the results are known as outcomes.

Random experiment

An experiment in which all possible outcomes are known, and the exact outcome cannot be predicted in advance, is called a random experiment.

Equally likely outcomes

If an outcome of an experiment is as likely to occur as the other, then such an outcome is called equally likely.

For example: Outcomes head and tail of the experiment "tossing a fair coin" are equally likely.

Event

A collection of one or more outcome (s) out of all possible outcomes of a random experiment is called its event.

An event having a single outcome is known as an elementary event, while an event obtained by combining two or more outcomes is called a compound event.

Sample space

A collection of all possible outcomes of a random experiment is known as the "sample space," which is represented by "S".

Occurrence of an event

An event 'E' associated to a random experiment is said to occur if any one of its outcomes is the result of the experiment.

Favorable outcomes

An outcome of an experiment is said to be favorable to an event 'E', if its occurrence implies the occurrence of events of event 'E'.

Theoretical probability (or) Probability

The probability of an event 'E'

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

Impossible event: The probability of an impossible event (while has no chance) is zero (0).

Sure event: An event which is sure to occur is called a "sure event" the probability of a sure event is 1.

Complementary event: An event (E) is said to be complementary of event E if $P(E) + P(\bar{E}) = 1$

MULTIPLE CHOICE QUESTIONS (1 MARK QUESTIONS)

1.A card is selected at random from a deck of 52 playing cards. The probability of it being a red face card is

(a) $\frac{3}{13}$ (b) $\frac{2}{13}$ (c) $\frac{1}{2}$ (d) $\frac{3}{26}$

Solution: Total number of possibilities = 5

Number of red face card = 06

Probability of red face card = $\frac{6}{52} = \frac{3}{26}$

Ans: (d) $\frac{3}{26}$

2.A Card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a spade (or) King?

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$ (c) $\frac{4}{13}$ (d) $\frac{9}{13}$

Solution: Total number of possibilities = 52

Number of spades= 13 (including King)

and 3 kings from clubs, diamonds & hearts

$\therefore$ P(spade(or) king) = $\frac{13+3}{52} = \frac{16}{52} = \frac{4}{13}$

Ans: (c) : $\frac{4}{13}$

3.The Probability that cannot exist among the following

(a) $\frac{2}{3}$ (b) -1.6 (c) 25% (d) 0.7

Solution: Probability cannot be negative

Ans: b): -1.6

4.The Probability that a number selected at random from the numbers 1,2,3, ---,15 is a multiple of 4 is

(a) $\frac{4}{15}$ (b) $\frac{2}{15}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3}$

Solution: Total number of possibilities = 15

Multiples of 4 are 4,8,12; (total 3)

P(multiples of 4) = $\frac{3}{15} = \frac{1}{5}$

Ans: (c): $\frac{1}{5}$

5.For an event E, if $p(E) + p(\bar{E}) = q$, then the value of $q^2 - 4$ is

(a) -3 (b) 3 (c)5 (d) -5

Solution: Maximum Probability= 1= q

$\therefore q^2-4 = 1^2- 4= -3$

Ans: (a) : -3

6.From the letters of the word "MOBILE", a letter is selected at random, the probability that the selected letter is a vowel is

(a) $\frac{3}{7}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

Solution: Total Possibilities =M+O+B+I+L+E=6

Number of vowels= O+ I +E =3

$\therefore$ Probability $= \frac{3}{6} = \frac{1}{2}$

Ans: (c): $\frac{1}{2}$

7.Two dice are tossed simultaneously. The probability of getting odd numbers on both dice is

(a) $\frac{6}{36}$ (b) $\frac{3}{36}$ (c) $\frac{12}{36}$ (d) $\frac{9}{36}$

Solution: Total number of Possibilities =36

The number of favorable outcomes is (1,1), (1,3), (1,5), (3,3),(3,5), (5,1), (5,3), (5,5) = 9

$\therefore$ Probability is $\frac{9}{36}$

Ans: (d): $\frac{9}{36}$

8.Two coins are tossed simultaneously. The probability of having exactly one head is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) None of these

Solution: Total no. of outcomes =4 (HH, HT, TT, TH)

Possibility of having one head= 2(HT, TH)

$\therefore$ Probability is $\frac{2}{4} = \frac{1}{2}$

Ans: (a): $\frac{1}{2}$

9. Three rotten eggs are mixed with 12 good ones. One egg is chosen at random. The probability of choosing a rotten egg is

(a) $\frac{1}{15}$ (b) $\frac{4}{5}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

Solution: Total number of eggs = 3+12=15=total possibilities

Number of rotten eggs= 3

$\therefore$ Probability is $\frac{3}{15} = \frac{1}{5}$

Ans: (c) $\frac{1}{5}$

10.17 cards are numbered as 1,2,3, ----------,17, then probability of divisible by 3 and 2 both

(a) $\frac{7}{17}$ (b) $\frac{5}{17}$ (c) $\frac{3}{17}$ (d) $\frac{2}{17}$

Solution: Total number of Possibilities =17

Divisible by both 3&2 means, multiple of 6

$\therefore$ Outcomes are 6,12

Probability is $\frac{2}{17}$

Ans: (d): $\frac{2}{17}$

ASSERTION AND REASONING QUESTIONS

DIRECTION: In the following questions, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

1.Assertion (A): In a simultaneous throw of a pair of dice. The probability of getting a doublet is $\frac{1}{6}$.

Reason (R): Probability of an event may be negative.

Ans: (c)

2.Assertion (A): The probability of winning a game is 0.4, then the probability of losing it, is 0.6.

Reason (R): P(E) + P (not E) = 1

Ans:(a)

3.Assertion (A): The probability of getting exactly one head in tossing a pair of coins is $\frac{1}{2}$.

Reason (R): The sample space of two coin tossed is = {HH, TT, HT, TH} = 4

Ans:(a)

4.Assertion (A): The probability that a leap year has 53 Sundays is $\frac{2}{7}$.

Reason (R): The probability that a non–leap year has 53 Sundays is $\frac{5}{7}$.

Ans. (c)

5.Assertion (A): The probability of getting a bad egg in a lot of 400 is 0.035. The number of good eggs in the lot is 386.

Reason (R): If the probability of an event is p, the probability of its complementary event will be 1-p.

Ans. (a)

6.Assertion (A): In a cricket match, a batsman hits a boundary 9 times out of 45 balls he plays. The probability that in a given ball, he does hit the boundary is $\frac{4}{5}$.

Reason (R):P(E) + P(not E) = 1

Ans. (a)

7.Assertion (A): Tanvi and Manvi were born in the year 2000. The probability that they have same birthday is $\frac{1}{366}$

Reason (R): Leap year has 366 days.

Ans. (a)

8.Assertion (A): A four-digit number is formed using the digits 1,2,5,6 and 8 without repetition. The probability that is an even number is $\frac{3}{5}$

Reason (R): The units digit of even number is also an even number.

Ans:(b)

9.Assertion (A): A dice is rolled. The probability of getting a composite number is $\frac{1}{3}$.

Reason (R): In a throw of a dice the probability of getting a prime number is $\frac{2}{3}$

Ans. (c)

10.Assertion (A): When a dice is rolled the probability of getting a number which is multiple of 3 and 5 both is 0.

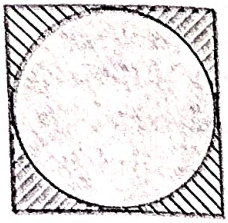
Reason (R): The probability of an impossible event is 0.

Ans. (a)

VERY SHORT ANSWER TYPE QUESTIONS(2 MARKS QUESTIONS)

1.There is a square board of side '2a' units circumscribing a yellow circle. Jayadev is asked to keep a dot on the above said board. Find the probability that he keeps the dot on the shaded region.

Ans. Area of square $= (2a)^2 = 4a^2$



Area of circle $= \pi r^2 = \pi a^2$

Difference $= 4a^2 - \pi a^2 = a^2(4 - p)$

Required probability = Favourable outcomes/Sample space

$\frac{a^2(4-p)}{4a^2} = \frac{(4-\pi)}{4}$

2.In an MCQ test, a student guesses the correct answer x out of y times. If the probability that the student guesses the answer to be wrong is 2/3 then what is the relation between x and y?

Ans. According to the given information, $P(\text{wrong}) = \frac{2}{3}$

The probability of guessing the correct answer is the compliment of the probability of guessing wrong answer.

$P(\text{correct}) = 1 - P(\text{wrong}) = 1 - \frac{2}{3}$

$P(\text{correct}) = \frac{1}{3}$

Now, the probability of guessing the correct answer P(correct) is the ratio of the number of correct (guesses) (x) to the total number of guesses (y)

$P(\text{correct}) = \frac{x}{y}$

$\therefore \frac{x}{y} = \frac{1}{3} \Rightarrow 3x = y \Rightarrow y = 3x$

3.A dice is thrown twice. Find the probability of getting 4, 5 or 6 in the first throw and 1, 2, 3 or 4 in the second throw.

Ans. Total number of outcomes on throwing a dice twice = 36

Here, favourable outcomes ={(4, 1), (4, 2),(4, 3), (4, 4),(5, 1), (5, 2),(5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4)}

∴ Number of favourable outcomes = 12

∴ Required probability $= \frac{12}{36} = \frac{1}{3}$

4. A school has five houses A, B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and the rest from house E. A single student is selected at random to be the class monitor. What is the probability that the selected student is not from houses A, B and C?

Ans. Total no. of students = 23

No. of students from houses A, B and C = 4 + 8 + 5 = 17

∴ Remaining no. of students = 23 – 17 = 6

∴ Required probability = No. of students, not from A, B and C / Total no. of students houses = $\frac{6}{23}$

5.A card is drawn from a well-shuffled deck of 52 playing cards. What is the probability that the card will not be an ace?

Ans. Total number of cards = 52

Number of non ace card = 52 – 4 = 48

∴ Required probability $= \frac{48}{52} = \frac{12}{13}$

6.A bag has 5 white marbles, 8 red marbles and 4 purple marbles. If we take a marble randomly, then what is the probability of not getting purple marble?

Ans: Total number of purple marbles = 4

Total number of marbles in bag = 5 + 8 + 4 = 17

Probability of getting not purple marbles $= \frac{13}{17} = 0.77$

7.Out of 200 bulbs in a box, 12 bulbs are defective. One bulb is taken out at random from the box. What is the probability that the drawn bulb is not defective?

Ans. Total no. of bulbs = 200

No. of defective bulbs = 12

No. of non-defective bulbs = 200 – 12 = 188

So, P (Getting a non-defective bulbs) $= \frac{188}{200} = \frac{48}{50}$

8.An integer is chosen at random between 1 and 100. Find the probability that it is:
(i) divisible by 8.
(ii) not divisible by 8.
Ans. (i) The integers divisible by 8 between 1 and 100 are 8, 16, 24, 32, 40, 48, 56, 64, 72. 80, 88, 96, i.e. 12
Total outcomes = 98
(i)P(divisible by 8) = $\frac{12}{98} = \frac{6}{49}$
(ii) P(not divisible by 8) = $\frac{98-12}{98} = \frac{86}{98} = \frac{43}{49}$
9.A ticket is drawn at random from a bag containing tickets numbered from 1 to 40. Find the probability that the selected ticket has a number which is a multiple of 5.
Ans: Total number of tickets = 40
Multiple of 5 are 5, 10, 15, 20, 25, 30, 35, 40
P(a number which is a multiple of 5) = $\frac{8}{40} = \frac{1}{5}$
10.Find the probability that a number selected at random from the numbers 1,2, 3,.....35 is a
Prime number ii) Multiple of 7
Ans:(i) Prime numbers are 2,3,5,7,11,13,17,19,23,29,31 Total-11 prime numbers
P(Prime) = $\frac{11}{35}$
(ii)Multiples of 7 are 7,14,21,28,35 Total-5 numbers
P(Multiple of 7) = $\frac{5}{35} = \frac{1}{7}$

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

1.A lot consists of 144 ball pens of which 20 are defective. The customer will buy a ball pen if it good, but will not buy a defective ball pen. The shopkeeper draws one pen at random from the lot and gives it to the consumer. What is the probability that
(i) customer will buy the ball pen (ii) customer will not buy the ball pen
Ans. Total ball pens = 144
Defective ball pens = 20
Good ball pens = 144 – 20 = 124
(i) Probability that the customer will buy the ball pen = $\frac{124}{144} = \frac{31}{36}$
(ii) Probability that the customer will not buy the ball pen = $\frac{20}{144} = \frac{5}{36}$
2. All the black face cards are removed from a pack of 52 playing cards. The reaming cards are well shuffled and then a card is drawn at random. Find the probability of getting (i) face card (ii) red card (iii) black card.
Ans: When all the black face cards are removed,
Remaining number of cards = 52 – 6 = 46
(i) Number of face cards in the remaining deck = 6
∴ P(getting a face card) = $\frac{6}{46} = \frac{3}{23}$
(ii) Number of red cards in the remaining deck = 26
∴ P (getting a red card) = $\frac{26}{46} = \frac{13}{23}$
(iii) Number of black cards in the remaining deck = 20
∴ P (getting a black card) = $\frac{20}{46} = \frac{10}{23}$
3. Two dice are thrown at the same time. What is the probability that the sum of the two numbers appearing on the top of the dice is
(i) at least 9? (ii) 7? (iii) less than or equal to 6?
Ans: (i) Number of outcomes with sum of the numbers is at least 9 = 10

∴ Required Probability $= \frac{10}{36} = \frac{5}{18}$

(ii) Number of outcomes with sum of the numbers $7 = 6$

∴ Required Probability $= \frac{6}{36} = \frac{1}{6}$

(iii) Number of outcomes with sum of the numbers less than or equal to $6 = 26$

∴ Required Probability $= \frac{18}{36} = \frac{5}{12}$

4. A bag contains 12 balls out of which x are white.

(*i*) If one ball is drawn at random, what is the probability that it will be a white ball?

(*ii*) If 6 more white balls are put in the bag, the probability of drawing a white ball will be double than that in (*i*). Find x.

Ans. $n(S) = 12$

(i)Let A be the event of drawing a white ball $n(A) = x$, $P(A) = \frac{x}{12}$

(ii)Number of white balls $= x + 6$

Total number of balls $= 12 + 6 = 18$.

Let B be the event of drawing a white ball

∴ $n(B) = x + 6$, $P(B) = \frac{x+6}{18}$

According to the question, $P(B) = 2P(A)$

$$\Rightarrow \frac{x+6}{18} = 2 \times \frac{x}{12}$$

$$\Rightarrow 6x + 36 = 18x$$

$$\Rightarrow 12x = 36 \Rightarrow x = 3$$

5. One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting

(i) a face card or a black card

(ii) neither an ace nor a king

(iii) a jack and a black card

Ans. Total number of playing cards $= 52$

(*i*) Favourable cases for a face card or a black card are 32 (12 + 26 – 6)

∴ Probability of drawing a king or a jack $= \frac{32}{52} = \frac{8}{13}$

(*ii*) Favourable cases for neither ace nor king card are 44 (52 cards – 4 aces – 4 king)

∴ Probability of drawing a non-ace $= \frac{44}{52} = \frac{11}{13}$

(*iii*) Favourable cases for jack and black card are 2

∴ Probability of drawing a red card $= \frac{2}{52} = \frac{1}{26}$

6.Two different dice are thrown together. Find the probability that the numbers obtained:

(a) have a sum less than 7

(b) have a product less than 16

(c) is a doublet of odd numbers.

Ans. The outcomes when two dice are thrown together, are:

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6),

∴ Total number of outcomes = 36

(a) Favourable outcomes are (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2) and (5, 1).

∴ Number of favourable outcomes = 15

∴ Required probability = $\frac{15}{36} = \frac{5}{12}$

(b) Favourable outcomes are (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1) and (6, 2).

∴ Number of favourable outcomes = 25

∴ Required probability = $\frac{25}{36}$

(c) Favourable outcomes are (1, 1), (3, 3) and (5, 5)

∴ Number of favourable outcomes = 3

∴ Required probability = $\frac{3}{36} = \frac{1}{12}$

7.A box contains 19 balls bearing numbers 1, 2, 3, ..., 19. A ball is drawn at random from the box. What is the probability that the number on the ball is

(i) a prime number

(ii) divisible by 3 or 5

(iii) neither divisible by 5 nor by 10

Ans. Total number of balls = 19

(i) Prime numbers from 1 to 19 are 2, 3, 5, 7, 11, 13, 17, 19 = Total 8 prime numbers

∴ Probability of drawing a prime number = $\frac{8}{19}$

(ii) Numbers divisible by 3 or 5 are 3, 6, 9, 15, 18, 10, 5, 12 = Total 8 numbers

∴ Probability of drawing a number divisible by 3 or 5 = $\frac{8}{19}$

(iii) Number divisible by 5 and 10 are 5, 10, 15 = Total 3

∴ Numbers which are neither divisible by 5 nor 10 are 19 – 3 = 16

∴ Probability of drawing a number which is neither divisible by 5 nor by 10 = $\frac{16}{19}$

8.Two dice are thrown simultaneously. What is the probability that

(a) 6 will not come up on either of them?

(b) 6 will come up on at least one?

(c) 6 will come up at both dice?

Ans: Total no. of outcomes = 36

(a) Number of outcomes in which 6 will not come up on either of them = 25.

∴ Required Probability = $\frac{25}{36}$

(b) Number of outcomes in which 6 will come up at least one die = 11.

∴ Required Probability = $\frac{11}{36}$

(c) Number of outcomes in which 6 will come up at both die = 1.

∴ Required Probability = $\frac{1}{36}$

9. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is: (i) red? (ii) not red?

Ans. Total number of balls = 3 + 5 = 8

(i) Number of red balls = 3

Hence, P (red ball) = $\frac{3}{8}$

(ii) Number of not red balls = 5

Hence, P (not red ball) = $\frac{5}{8}$

10. An integer is chosen at random between 1 and 100. Find the probability that it is:

(i) divisible by 8.

(ii) not divisible by 8.

Ans. (i) The integers divisible by 8 between 1 and 100 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, i.e. 12

Total outcomes = 98

P(divisible by 8) = $\frac{12}{98} = \frac{6}{49}$

(iii) P(not divisible by 8) = $\frac{98-12}{98} = \frac{86}{98} = \frac{43}{49}$

LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1. A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One piece is lost at random. Find the probability that it is a

(i) triangle (ii) square (iii) square of blue colour (iv) triangle of red colour

Ans: Total number of pieces = 8 + 10 = 18

(i) No. of triangles = 8, Hence, P(triangle is lost) = $\frac{8}{18} = \frac{4}{9}$

(ii) No. of squares = 10, Hence, P(square is lost) = $\frac{10}{18} = \frac{5}{9}$

(iii) No. of squares of blue colour = 6, So, P(square of blue colour is lost) = $\frac{6}{18} = \frac{1}{3}$

(iv) No. of triangles of red colour = 8 – 3 = 5, So, P(triangle of red colour is lost) = $\frac{5}{18}$

2. From a pack of 52 playing cards, jacks, queens, kings and aces of red colour are removed. From the remaining a card is drawn at random. Find the probability that the card drawn is (i) a black queen (ii) a red card (iii) a face card (iv) a spade card

Ans: From the total playing 52 cards, red coloured jacks, queen, kings and aces are removed(i.e., 2 jacks, 2 queens, 2 kings, 2 aces) ∴ Remaining cards = 52 – 8 = 44

(i) Favourable cases for a black queen are 2 (i.e., queen of club or spade)

∴ Probability of drawing a black queen = $\frac{2}{44} = \frac{1}{22}$

(ii) Favourable cases for red cards are 26 – 8 = 18 (as 8 cards have been removed) (i.e.9 diamonds + 9 hearts)

∴ Probability of drawing a red card = $\frac{18}{44} = \frac{9}{22}$

(iii) Favourable cases for a face card are 6 (i.e. 2 black jacks, queens and kings each)

∴ Probability of drawing a face card = $\frac{6}{44} = \frac{3}{22}$

(iv) Favourable cases for a spade card are 13

∴ Probability of drawing a spade card = $\frac{13}{44}$

3. A box contains cards bearing numbers from 6 to 70. If one card is drawn at random from the box, find the probability that it bears

(i) a one digit number. (ii) a number divisible by 5.

(iii) an odd number less than 30. (iv) a composite number between 50 and 70.

Ans: Number of cards in the box = 65

(i) Cards bearing one digit numbers are 6, 7, 8, 9

Number of such cards = 4

∴ Probability of card bears a one digit number = $\frac{4}{65}$

(ii) B = Number on the cards is divisible by 5
∴ Cards favourable to B are 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70
∴ $P(B) = \frac{13}{65} = \frac{1}{5}$
(iii) C = Cards with an odd number less than 30 i.e. 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29
$P(C) = \frac{12}{65}$
(iv) D : Card with composite number between 50 and 70
i.e. 51, 52, 54, 55, 56, 57, 58, 60, 62, 63, 64, 65, 66, 68, 69
∴ $P(D) = \frac{15}{65} = \frac{3}{13}$

4. From a pack of 52 playing cards, Jacks and Kings of red colour and Queens and Aces of black colour are removed. The remaining cards are mixed and a card is drawn at random. Find the probability that the drawn card is:
(a) a black queen.
(b) a card of red colour.
(c) a Jack of black colour.
(d) a face card.

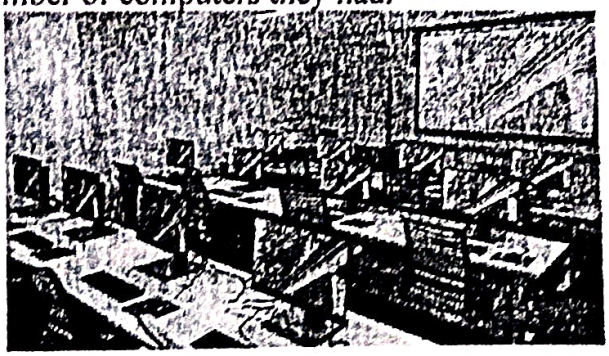
Ans. Number of cards removed = (2 + 2 + 2 + 2) = 8
Total number of remaining cards = (52 – 8) = 44
Now, there are 2 jacks, 2 kings of black colour and 2 queens, 2 aces of red colour left.
(a) Number of black queens = 0
∴ P(getting a black queen) = $\frac{0}{44} = 0$
(b) Number of red cards = 26 – 4 = 22
∴ P(getting a red card) = $\frac{22}{44} = \frac{1}{2}$
(c) Number of jacks of black colour = 2
∴ P(getting a black jack) = $\frac{2}{44} = \frac{1}{22}$
(d) We know that jacks, queens and kings are face cards.
∴ Number of remaining face cards = (2 + 2 + 2) = 6
∴ P(getting a face card) = $\frac{6}{44} = \frac{3}{22}$

5.Red Queens and black jocks are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards, after reshuffling them. Find the probability that the drawn card is
(i) a king (ii) red colour (iii)a face card (iv)a queen

Ans: i) $\frac{1}{12}$ ii) $\frac{1}{2}$ iii) $\frac{1}{6}$ iv) $\frac{1}{24}$

CASE BASED QUESTIONS (4 MARKS QUESTIONS)

1.Computer Based Learning (CBL) refers to any teaching methodology that makes use of computers for information transmission. At an elementary school level, computer applications can be used to display multimedia lesson plans. A survey was done on 1000 elementary and secondary schools of Assam and they were classified by the number of computers they had.



Number of Computers	1 – 10	11 – 20	21 – 50	51 – 100	101 and more
Number of Schools	250	200	290	180	80

One school is chosen at random.
Based on the given information, solve the following questions:
(i) Find the probability that the school chosen at random has more than 100 computers.
(ii) Find the probability that the school chosen at random has 50 or fewer computers.

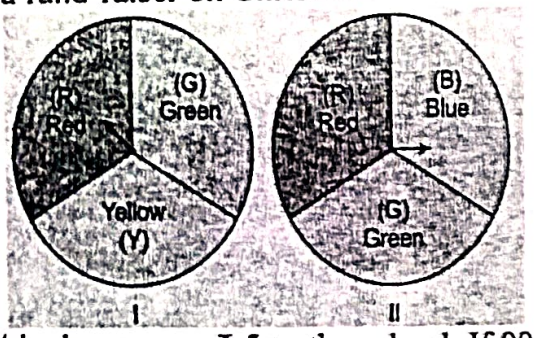
(OR)

Find the probability that the school chosen at random has no more than 20 computers.
(iii) Find the probability that the school chosen at random has 10 or less than 10 computers.
Ans. (i) Total number of possible outcomes = Number of schools = 1000
Number of favourable outcomes = Number of schools has more than 100 computers = 80
∴ Required probability = $\frac{80}{1000} = \frac{2}{25}$
(ii) Total number of possible outcomes = Number of schools = 1000
Number of favourable outcomes = Number of schools has 50 or fewer computers = 250 + 200 + 290 = 740
∴ Required probability = $\frac{740}{1000} = \frac{37}{50}$

(OR)

Total number of possible outcomes = Number of schools = 1000
Number of favourable outcomes = Number of schools has no more than 20 computers = 250 + 200 = 450
∴ Required probability = $\frac{450}{1000} = \frac{9}{20}$
(iii) Total number of possible outcomes = Number of schools = 1000
Number of favourable outcomes = Number of schools has 10 or less than 10 computers = 250
∴ Required probability = $\frac{250}{1000} = \frac{1}{4}$

2.A middle school decided to run the following spinner game as a fund-raiser on Christmas
Making Purple: Spin each spinner once. Blue and red make purple. So, if one spinner shows Red (R) and another Blue (B), then you 'win'. One such outcome is written as 'RB'.
Based on the given information, solve the following questions:



(i) List all possible outcomes of the game.

(ii) Find the probability of 'Making Purple'.

(iii) For each win, a participant gets ₹ 10, but if he/she loses, he/she has to pay ₹ 5 to the school. If 99 participants played, calculate how much fund could the school have collected.

(OR)

If the same amount of ₹ 5 has been decided for winning or losing the game, then how much fund had been collected by school? (Number of participants = 99)
Ans. (i) Spinner I Spinner II
Red (R) – Red (R) (RR)
Red (R) – Blue (B) RB,
Red (R) – Green (G) RG,
Green (G) – Red (R) GR,
Green (G) – Blue (B) GB,
Green (G) – Green (G) GG,
Yellow (Y) – Red (R) YR,
Yellow (Y) – Blue (B) YB,
Yellow (Y) – Green (G) YG}
Total number of outcomes = 9
(ii) X = {RB}